Estimating the Reliability of Hydraulic Pumps and Motors under Consideration of Different Loads

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Abstract

The goal of this paper is to present a method to provide quantitative predictions for the reliability of products and parts using given information about failure behavior and about respective loads. The required previous knowledge is accessible either due to previous generations of the product, or it can be generated via analysis experience of the product in the field. The focus of this work is on the concept of a new method in general as well as on the detailed description of the method in the special case of fatigue calculation.

KEYWORDS: reliability, Weibull, Weibayes, accelerated life testing, fatigue

1. Introduction

Hydraulic drive systems are facing a general trend leading towards the construction of more and more efficient drive mechanisms. Higher degrees of efficiency, smaller construction spaces, lighter weights and higher system pressures are the main foci with respect to this development. However, the resulting increase in power density and complexity of the systems also leads to more complex failure mechanisms. The proper handling of these mechanisms requires a well-functioning reliability management. In order to determine the reliability of systems or their components, they are normally tested in test stand trials before starting series production. Attention should be paid to the fact that the statistical uncertainty because of the testing of few units is relatively high. For the planning of lifetime tests, test specimen size and the linked factor of level of confidence are crucial parameters. In general, a preferably high number of test units is desirable. However, a high number of test units causes an increase in duration and costs.

2. Reliability

Reliability is the probability that a product does not fail under given functional und environmental conditions during a defined period of time /1/. Quantitatively, the failure mechanisms of components or systems can only be described by probabilities. This is due to the fact that failure mechanisms are coincidentally stochastically distributed. In fact, it is very unlikely that several components which are randomly selected from production will simultaneously fail while encountering identical loads. The time up to the failure of a single component is subject to a certain statistical spread. In contrast, the lifetime distribution of the population stays constant under controlled conditions /1, 2/. This lifetime distribution has been described frequently by engineers using the Weibull distribution.

2.1. Weibull distribution

The Weibull distribution is a very adaptable mathematical distribution function, which basically describes the probability distribution of a real random variable /1, 3/. The Weibull distribution has proved to be very useful to mechanical engineering in accounting for the failure behavior of mechanical components. The Weibull distribution is defined by the shape parameter b, the characteristic lifetime T and the parameter $t0$, needed for a three parametric distribution. The shape parameter b determines the failure slope and thus has an influence on the shape of the Weibull-curve. The shape parameter being 1 (b=1) results in an exact exponential distribution.

The normal distribution approximately corresponds to the Weibull distribution with a shape parameter of b=3.5. The characteristic lifetime T is a type of mean value, which is constantly $F(t)$ = 63.2 %, if t=T. The Density Function $f(t)$ and the failure probability $F(t)$ of the Weibull distribution can be described by (1) and (2) /1/.

$$f(t) = \frac{b}{T-t_0} * \left(\frac{t-t_0}{T-t_0}\right)^{b-1} * e^{-\left(\frac{t-t_0}{T-t_0}\right)^b} \quad (1)$$

$$F(t) = 1 - e^{-\left(\frac{t-t_0}{T-t_0}\right)^b} \quad (2)$$

Figure 1 shows the Weibull functions with varying shape parameter b.

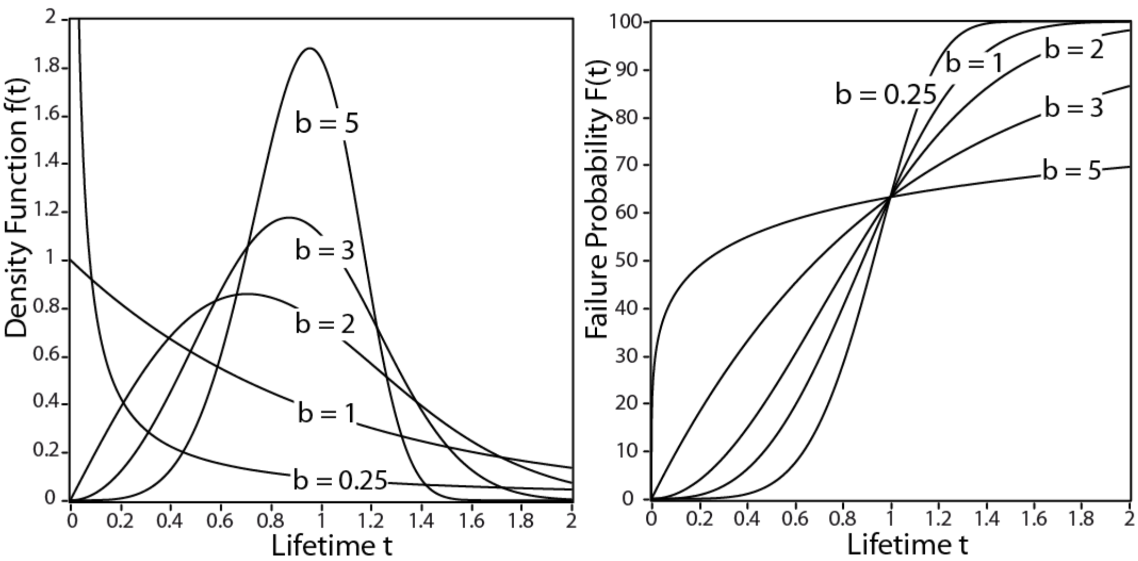

Figure 1: Density function $f(t)$ and failure probability $F(t)$ of the Weibull distribution

2.2. System reliability

In order to determine the failure behavior of systems, the failure behavior of each component has to be identified. Assuming that components are non-repairable or that the first failure is decisive for the system, the components can only be categorized as “functional” or “failed”. If, in addition, the failure mechanisms cannot influence one another, the system reliability can be calculated using the Boolean Theory.

There are two structures that can cause a failure of the whole system. The first is the serial structure, in which the failure of one single component leads to a failure of the whole system. The second is the parallel structure, in which all components have to fail in order to cause a failure of the whole system. In this context, a system can consist of any number of series or parallel structures. Since redundancies are expensive and complex, systems mostly consist of pure serial structures and can thus be defined quite simply in mathematical terms, as in (3) /1/:

$$F(t) = 1 - \prod_{i=1}^{n}(1 - F_i(t)) \tag{3}$$

Using the Weibull distribution as the input parameter for single components, an exact Weibull distribution for the system will be received only in exceptional cases. Since the Weibull distribution has a high universality, often a very good approximation function can be determined. **Figure 2** shows that the Weibull distribution of the failures in a system with a serial structure strongly depends on the system's weakest components. Thus, it can be sufficient to determine the failure behavior of the components which have the highest failure rates and then to derive the system reliability from these values. However, one has to take into account that this is a best-case consideration.

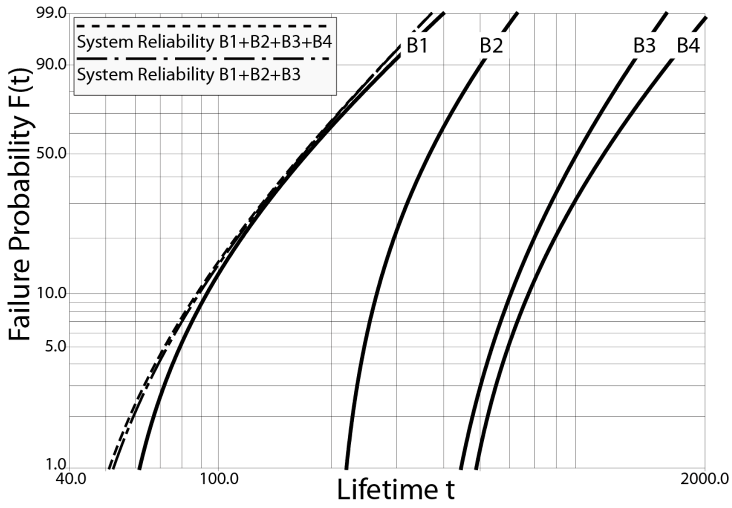

Figure 2: Weibull distribution of a system with a serial structure

2.3. Test planning to evaluate a required minimal lifetime

Practically, test planning is usually conducted based on the binomial distribution, which depicts a discrete probability distribution. Using the binominal distribution, the probability for the outcome of a random experiment can be determined - the results being mutually exclusive. According to /4/, the probability function of the binomial distribution for the event A can be specified as:

$$f(x) = P(X = x) = \binom{n}{x} p^x \cdot q^{n-x} \quad (x = 0, 1, 2, \dots, n) \tag{4}$$

A practical request would be to calculate the minimal reliability *R(t)* with respect to a certain lifetime and a given confidence level. Assuming a minimally required B10 lifetime of 1000 hours with a one-side confidence coefficient of 95% (*R*(1000h)>=90% with *PA*=95%), would mean that 29 components have to be tested for 1000 hours. Using the lifetime ratio, it is possible to correct the number of test units with respect to the test period. If a Weibull distribution is assumed, it is possible to calculate the number of required test items including the lifetime factor (5) /1/.

$$R(t) = (1 - P_A)^{\frac{1}{L_v{}^b \cdot n}} \quad (5)$$

2.4. Determination of the lifetime distribution

In order to calculate a reliability distribution, the failure times of the respective components or of the respective system have to be known. It is most simple to generate a reliability distribution if the product has already been applied in the field and real failure times are known. It is more challenging estimate reliability distribution before the product comes onto the market. The best option is to test a certain number of test units under representative conditions until they fail. In doing so, the long-running test units, whose lifetime is heading towards the end of the lifetime scale, especially exploit a lot of resources. It is recommended to interrupt the test stand trials before all test units have failed. With respect to Censoring Type I, all test stand trials are to be canceled after a certain amount of time. Test stand trials which are canceled after a given number of failed test units are labeled Censoring Type II. The lifetime distribution thus determined is based on failure times, which underlie a certain statistical spread. Thus, the calculated lifetime distribution is, on average, the most probable one. However, deviations with respect to both directions are possible. This is why a confidence level has to be specified, in order to consider the uncertainty in lifetime distribution. The confidence level of 90% depicted in **Figure 3** shows that the failure of a random test specimen can be assumed to be within the confidence limits with a probability of 90%. The range covered by the confidence interval becomes thinner with an increase in test specimen size.

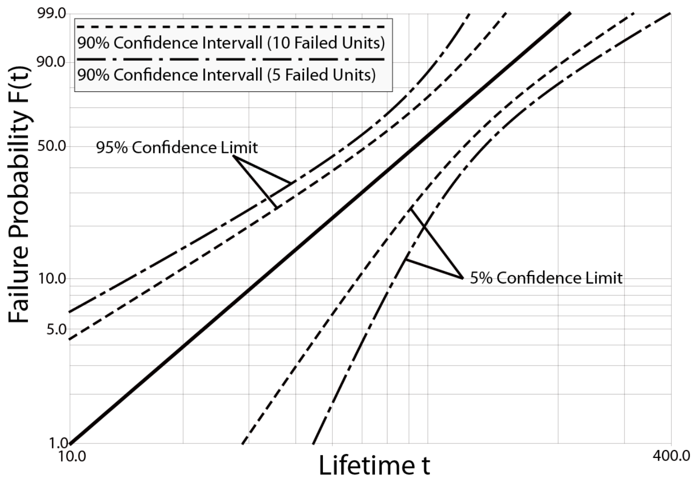

Figure 3: Confidence interval of a Weibull distribution

3. Accelerated testing

Ideally, a high number of components has to be tested for every failure mechanism, in order to be able to make statistically firm statements with respect to the expected reliability of components. There are some strategies for shortening the test time of lifetime tests in order to limit the test stand trials, which are linked directly to high financial and material costs. There are different types of approaches: on the one hand, there are strategies that can be applied in order to reduce the test period. These methods can evoke failures via higher loads. On the other hand, there are strategies that can be applied in order to reduce the test period by reducing the test specimen size.

4. Strategies for shortening the test period

The most common strategies for shortening test periods are /1, 5/:

- Accelerated testing at elevated stress levels
- Step-Stress-Method
- Degradation Test
- HALT – Highly Accelerated Life Testing
- Consideration of Prior Information (Bayesians-Method)

For all those strategies, there are common limitations:

- The failure mechanism does not change /1, 5/.
- The accuracy of the results of accelerated lifetime tests is inversely proportional to the time-acceleration factor of the trials. /5/
- The existing strategies are validated and verified by examples, most of the time. A general validation often is not possible. Thus, a critical judgement of the results is inevitable. /1/
- If a product is affected by several failure mechanisms, those have to be investigated separately.

These limitations, however, only hold if adequate failure times are required. The handling of the above-mentioned strategies is less problematic if only the existing failure mechanisms are to be identified.

5. Prediction of the reliability characteristic depending on different load spectrums

Very different application-specific loads may occur with respect to hydraulic pumps and motors. This leads to the consequence that the lifetime distribution should be determined separately, not only for each failure mechanism, but also for each load. The concept presented in this paper aims to derive the failure behavior with respect to a certain load via known failure behavior by a comparison with the known load spectrum. If no load spectrum with a familiar failure behavior exists, a standard load spectrum, which can be used as a reference, has to be defined. In order to link the information from the reference load spectrum with the relative damage of the load spectrums from the lifetime calculation, the Weibayes method can be applied. With the Weibayes method, the characteristic lifetime and thus all required parameters for the Weibull distribution of the failures in (6) /2/ can be estimated with a small effort, using the known Weibull parameter b /2/.

$$T = \left[\sum_{i=1}^{N} \frac{{t_i}^b}{r}\right]^{\frac{1}{b}} \qquad (6)$$

The form parameter b can be assumed to be constant in a first approximation for differing load spectrums with identical failure mechanisms since failure statistics have shown that the form parameter is extensively constant for identical types of failure /1, 2/. Adding a term which connects the characteristic lifetime of the distribution assumed to be familiar with a failure parameter to (6), the formula (7) can be received. The characteristic lifetime of the yet unknown lifetime distribution can be calculated using this formula.

$$T = \left[\frac{\alpha \cdot {t_R}^b}{\alpha + r} + \left(\sum_{i=1}^{N} \frac{{t_i}^b}{\alpha + r}\right)\right]^{\frac{1}{b}} \qquad (7)$$

This formula offers, with t_i and r, the possibility to add real failure times from test stand trials or field tests to the calculated parameters, in order to support the results received for the lifetime distribution. If no real failure data exist, t_i and r have to be equated with 0. Using the parameter α, the influence on the result by the calculation in comparison to real failure times can be weighted.

5.1. Determination of the failure parameter using fatigue analysis techniques

Lifetime calculation is often put on a par with fatigue calculation. Depending on the actual instance of failure, however, one has to take into account which concrete failure mechanism is the reason for the actual failure. Wear or an aggressive medium can, for example, cause failure as well. The strategy for the determination of the damage parameter using fatigue calculation presented in this paper is only one approach. The determination of the failure parameter also assumes a major comprehension of failure mechanisms, since incorrect assumptions with respect to the connection between load and failure lead to wrong results.

5.2. Fatigue calculation according to the notch stress approach

The notch stress approach is, as almost all fatigue analysis techniques, based on a comparison of the occurring load and the tolerable loads /7/. The method described in this paper uses the finite element method in order to determine the occurring load. The advantage of this method is that reliability can be estimated early in the development process. Several steps are necessary in order to compare the stresses received from the FEM with the S-N Curve. These steps are explained in the following.

5.2.1. Calculation of the occurring stresses with the finite element method

The finite element method is a technique used in order to approximately solve partial differential equations which can provide the static stresses at any number of nodes /6/. Assuming a linear-elastic behavior, the stress tensor for any point in time can be calculated by scaling the respective unit load. Therefore the given load spectrum is used /7/. If mutually independent stresses affect a certain unit, the stress tensors can be scaled and accumulated according to the superposition method (8) /7/.

$$e_{\sigma_{ij}}(s,t) = \sum_k c_{ij,k}(s) \cdot L_k(t) \tag{8}$$

5.2.2. Determination of a measurement for requirement stresses

There are different hypotheses on how to calculate equivalent stresses in order to define a measurement for the requirement stresses from this multiaxial stress. Two example equivalent stress hypotheses are:

- The maximum principal stress hypothesis (Lamé, Rankine)
- The von Mises stress hypothesis (Huber, von Mises, Hencky)

The von Mises stress hypothesis only provides positive values. According to /7/, the algebraic sign from the maximum principal stress has to be used.

5.2.3. Classification of the data

With respect to the fatigue calculation, only the number of load cycles, their amplitudes and mean loads have to be taken into account. Thus, it is expedient to evaluate the load spectrums with statistical counting methods in order to reduce the data volume as much as possible. Considering this, the rain flow counting method /7/ can be used. It provides the ideal base with respect to the fatigue calculation by carrying out a comparison with the respective S-N Curve.

5.2.4. Accounting for the mean stress influences

With the Haigh-curve the influence of the stress ratio and the mean stress respectively can be calculated /7/. This has to be done if a comparison with the S-N Curve, which has usually been generated at a changing level stress ratio (R=-1), is desired. In the area of R –inf and 0 the falling gradient is given with the value of the mean stress responsivity M. In the area for R between 0 and 0.5, the gradient is M/3 /7/.

5.2.5. Linear damage accumulation hypothesis

The previous calculations were carried out in order to find an equivalent load by means of a sinusoidal load with varying amplitudes for the damage caused by the load spectrum. In doing so, the stresses arising in the S-N Curve were made comparable. The S-N Curve represents the fatigue strength of components or materials. The underlying FEM calculation leads to the use of the material S-N curves. These empirical material S-N curves, whose validity is supported by trials, can be found in the corresponding literature (for example, FKM /8/). In order to estimate the lifetime, the ratio between load cycle number n and load cycles to failure N for any amplitude can be calculated and summed up /7/. Thus, a damage sum for each node, from which the maximal value for the prediction of the failure depends, can be received. If the damage sum reaches the value S=1, one has to expect a failure, according to Miner /7/. However, this assumption often does not hold and the absolute failure value should be determined via trials. Searching for an input parameter for the estimation of the reliability for varying load spectrums, as described in this paper, the problem does not occur any more. Only the relative failure between the respective load spectrums is needed. **Figure 4** depicts the fatigue calculation with respect to a fictitious example:

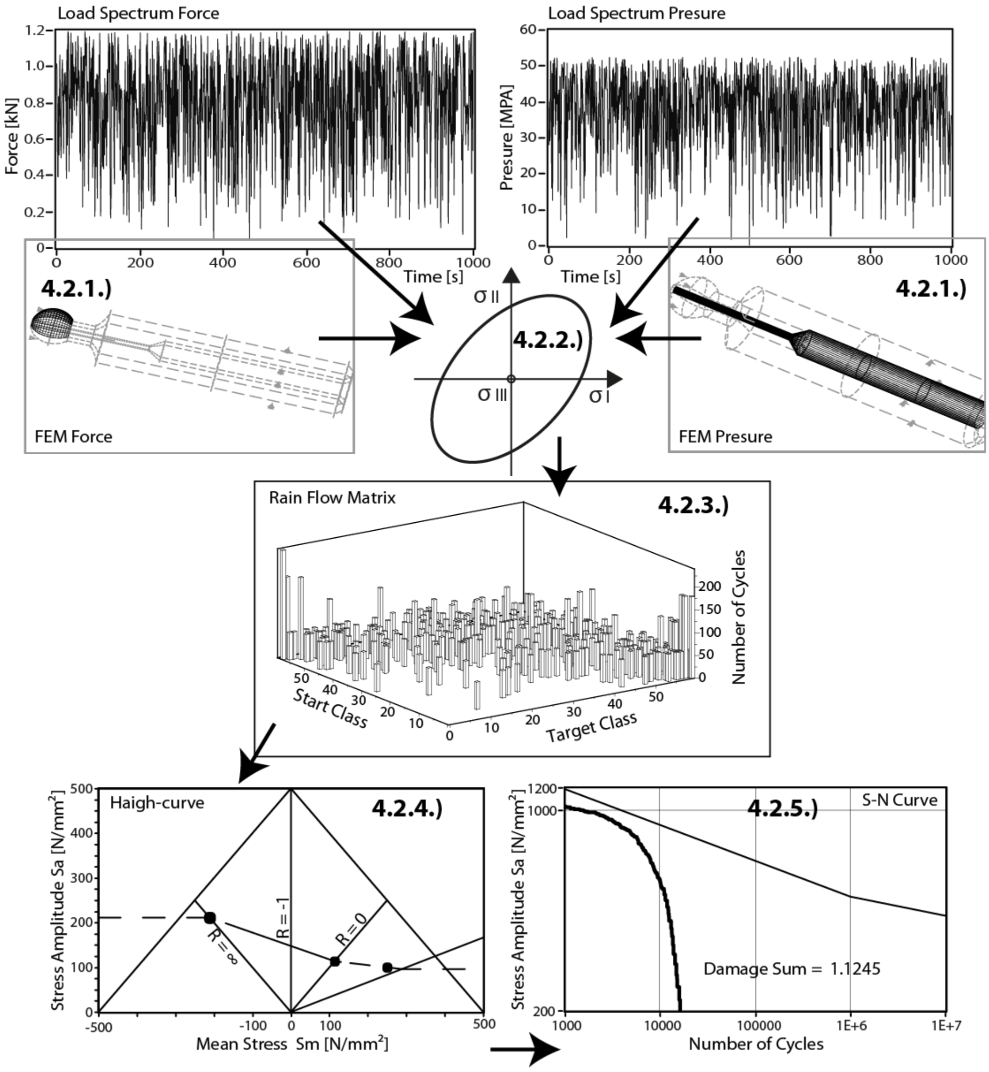

Figure 4: Fatigue calculation

6. Practical example

Figure 5 shows three lifetime distributions according to Weibull for one component of a hydraulic pump. The lifetime distribution 1 with the underlying given load spectrum 1 was determined on the basis of field tests, and can thus be assumed to be familiar. The lifetime distribution 2 was determined on the base of the fatigue calculation for a further application with a diverging load spectrum using the failure parameter. In addition, the lifetime distribution 3 shows the real lifetime distribution which results from field tests after the introduction of the new application.

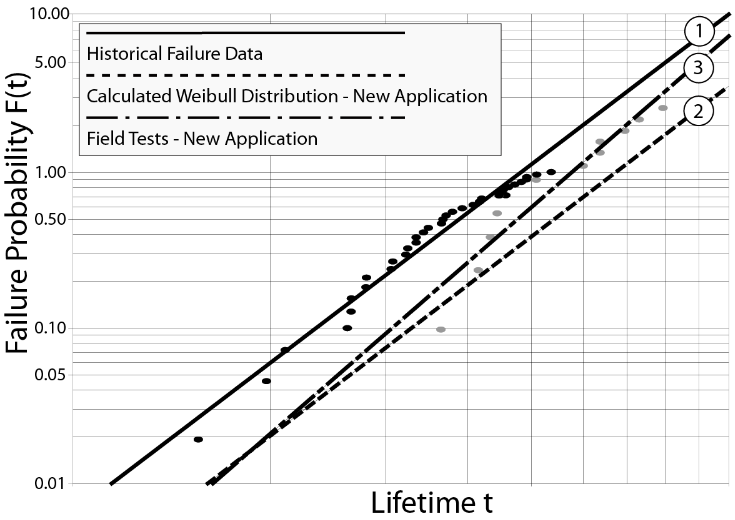

Figure 5: Calculated Weibull distribution

7. Conclusion

Since the technical requirements of hydraulic units are continuing to increase, considering the reliability of the products in detail has become unavoidable. In the ideal case, one had to test a multitude of units for each failure mechanism, in order to make statistically valid judgments about the failure behavior of a system for a given application. The concept described in this paper explains how the reliability parameters can be estimated in an early stage of the development and also under changing loads. However, it should not be forgotten that each instance of shortening of the testing time leads to a residual risk with respect to the informative value of the results. Looking at a real example it is easy to see that the assumption of identical form parameters *b* is only conditionally permitted. A deviation of the characteristic lifetime *T* can also be identified. The concept described in this paper should not be understood as an alternative to test stand trials, but as an extension for evaluation of new products and applications. Any information concerning the failure behavior from test stand trials or from field data advances the calculationally-supported lifetime distribution. The best strategy is to combine the field experience of existing products with the simulation and lifetime calculation of new products or new applications.

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9. Nomenclature

α	Weighting factor	-
σ	Stress tensor	-
b	Shape parameter	-
c	Proportionality factor	-
Lk	Single load	N
Lv	Lifetime factor	-
n	Number of test items	-
P	Probability	%
r	Number of failed units	-
s	Stress	N/mm²
T	Characteristic lifetime	-
t	Time	s
$t0$	Failure free time	-
tR	Calculated lifetime	-