# Modeling and Simulation of Pneumatic Systems with focus on Tubes

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#### Abstract

The design and dimensioning of pneumatic applications is currently done at Festo AG & Co. KG with simulation programs such as CACOS<sup>®</sup> requiring dynamic models for each component that represent both its steady-state and transient behaviour. The effects of pneumatic tubes connecting system components have been neglected so far to increase computational speed and reduce model size. In process automation, one of the novel application fields at Festo, long pneumatic tubes are used to cover the distance between distributed drive units and centrally arranged valve clusters. The influence of these tubes on the overall system dynamics is relevant and worth to be studied. Therefore, a dynamic tube model with a good trade-off between model complexity and accuracy is derived. It is designed to be compatible with signal-flow simulation programs, increasing its usability in large-scale industrial settings. The model is validated against experimental results for pressurizing and exhaust processes for tubes between 2 and 200m.

KEYWORDS: modelling, simulation, pneumatic system, tubes, transient behavior

#### 1. Introduction

Standard pneumatic drive systems are split into the following subsystems: pneumatic cylinders as positioning units, proportional or directional control valves for adjusting mass flow, and pressure regulating units. Simulation programs are used at Festo AG & Co. KG for design and dimensioning of pneumatic applications, aiding the choice of the components in size and type, the computation of controller parameters and allowing more accurate prediction of energy costs. The effects of pneumatic tubes connecting system components and their representation as dynamic models have been neglected so far to increase computational speed and reduce model size. Especially, since simulation programs run web-based with direct access for customers. With the novel

application of process automation, the influence of tube dynamics on the overall system such as delayed pressure increase at rear ends is of interest. Controllers have to be adapted to account for these dynamics, if possible with the help of pre-simulations. Hence, the modeling of mass flow and pressure over time and space for pneumatic tubes is getting more important (called distributed parameter modeling). Tubes models based on the well-known C-b-method with approximations for the effects of tube inner volume are not sufficient anymore due to their inaccuracy /1/.

Tube with diameter D [m] and length L [m]  

$$p_{high} \xrightarrow{\{C,b\}} p_{low}$$
Approximation formula by /1/  
 $C = \frac{0.029D^2}{\sqrt{L/D^{5/4} + 510}}, \quad b = \frac{474 \cdot C}{D^2}$ 

Calculation of mass flow with flow function

$$\dot{m} = \rho \cdot p_{high} \cdot C \cdot \Psi\left(p_{low}/p_{high}, b\right) \quad \rightarrow \quad \Psi\left(q = \frac{p_{low}}{p_{high}}, b\right) = \begin{cases} 1 & , q < b \\ \sqrt{1 - \left(\frac{q - b}{1 - b}\right)^2} & , q \ge b \end{cases}$$

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# **Fig. 1:** Formula for calculation of mass flow through a pneumatic resistance with diameter D and length L in [meters] ([m]).

The dynamic modeling and numerical implementation strategies for general fluidic transmission lines have been studied thoroughly, mainly for application in fluidic pipeline networks: leak detection in oil pipelines or design optimization of water networks. Compared to that, the detailed, theoretical analysis of pneumatic networks has not been a priority so far /2/. Recent interest in energy efficiency has seen a resurgence of these fields for the overall compressed air infrastructure, with impact on manufacturing processes, simulation of pneumatic drive components, and other theoretical analysis applications. Aspects such as leakage losses, dimensioning of tubes and networks and low-energy generation are currently under active research /3/. This paper gives a short summary of existing modeling and implementation strategies for fluidic transmission lines. Most approaches for water or oil pipeline network applications consider laminar flow. Hence, the dynamic equations can often be solved analytically, e.g. by series approximations. But considering fluidic tubes used in drive applications, turbulent flow conditions can't be neglected. Additionally, the compressibility of air is an essential characteristic in pneumatic applications. A loworder model is derived that assumes non-linear turbulent flow, and hence the equations have to be solved numerically since an analytic solution does not exist. It is designed to be compatible with signal-flow oriented simulation programs, increasing its usability in large-scale industrial settings. The model is numerically well-conditioned for tubes between 2 and 200m and all parameters are determined by physical or empirical relationships. It is finally validated against experimental results for inflation and exhaust processes.

The paper is structured as follows: Section 2 shows the derivation of the dynamic model for pneumatic tubes with a summary of fluidic transmission line modeling, the reformulation of the general equations into ordinary differential equations (Section 2.1) and a discussion of friction term modeling (Section 2.2). A thorough validation of the model equations against measurements is given in Section 3. Section 4 deals with a general background on application possibilities of the model from practice. A conclusion is given in Section 5.



#### 2. Dynamic model for long pneumatic tubes

Fig. 2: Inflation process of a pneumatic tube with inner diameter D = 5.7mm and length L =  $\{5,100\}$ m.

The measurements in **Fig. 2** show in the inflation of a pneumatic tube starting at ambient conditions. Three of four main dynamic characteristics are visible: at beginning of pressurizing process, dead time  $\Delta t=L/v_{ss}$  is identifiable, that results from limited speed of pressure waves in air (compressibility) with  $v_{ss}=343$  m/s. Pressure rise does not start before pressure waves reach end of the tube. The inner volume of the tube leads to a delayed rise of pressure at the rear end. The pressure waves are reflected at the end of the tube which is clearly visible for shorter tubes. Hence, also the mass flow shows oscillating behavior with damping effect for longer tubes. Not visible here is the pressure drop over the tube related to a constant flow rate through the tube, resulting

from wall stress. A good overview of general fluidic transmission line modeling is given in /4/. The approach for pneumatic transmission lines given in Section 2.1 is based in the following model requirements: (a) the dynamic equations are ordinary differential equations of first order not linked to a special simulation software, (b) model parameters are standard industrial parameters or empirically determined and physically explainable; elaborate parameters estimation has to be avoided, (c) simulation is not supposed to be slowed down drastically by the integration of the model and (d) all four main characteristics have to be pictured by the model.

#### 2.1 Basic equations

The basic equations distributed parameter modeling of fluidic transmission lines, i.e. pressure and flow rate over space and time, are taken out of canonical studies on fluid mechanics. The set of formulas needed to derive the dynamic equations consists of the continuity equation that states that the amount of mass inside a tube can only change by the amount of mass that passes through its rear ends (no leakage), the three dimensional Navier-Stokes equations (here: one-dimensional, axis parallel flow) and some general formulas such as the general law for ideal gases and the empiric viscosity equation after Sutherland which is important for compressible fluids and accounts for surrounding temperature. Based on these equations, three main models for general fluidic transmission lines /5/ are found in the literature - the lossless line model /8/, the average friction model and the distributed parameter model /9/. After detailed study of these three models and their different approaches of handling friction and loss effects, the average friction model is chosen for pneumatic transmission lines. The general equations are changed by replacing head by pressure and by adding a pressure and temperature dependent density for compressible fluids. The equations for the average friction model are given in general as coupled, partial hyperbolic differential equations for p(t,x) and m(t,x) with A = flow cross section, T = temperature and R= air gas constant

$$-\frac{\partial p}{\partial x} = \frac{1}{A}\frac{\partial \dot{m}}{\partial t} - f_{nl}(\dot{m}, p), \qquad -\frac{\partial \dot{m}}{\partial x} = \frac{A}{RT}\frac{\partial p}{\partial t} \implies p(x, t), \dot{m}(x, t)$$
(1)

There exist different solution strategies for partial differential equations. Due to the nonlinear and coupled structure of Eq. (1), an analytical solution is too difficult to find and the equations have to be solved numerically to be implemented for simulations. Therefore, a semi-discretization approach, i.e. vertical line method, with a spatial discretization into n tube segments is performed. The variables pressure and mass flow are placed on the spatial grid by the control-volume method /10/ shown in **Fig. 3** (a).

The dynamic equations as ordinary differential equations in time for the i-th out of n elements are written as follows:

$$\frac{d\dot{m}_i}{dt} = \frac{\mathsf{A}}{\Delta x} \left( p_{i-1} - p_i \right) - f_{nl}(\dot{m}, p) \quad \frac{dp_i}{dt} = \frac{\mathsf{RT}}{\mathsf{A}\Delta x} \left( \dot{m}_i - \dot{m}_{i+1} \right) \tag{2}$$

As shown in Fig. 3 (a), the model is presented as a two-port system with two inputs and two outputs representing the values of pressure and flow rate at the rear ends of the tube. There are four combinations of boundary conditions possible, two are presented here. They are dependent on the elements connected to the pneumatic tube model, i.e. when the tube is connected to two volumes; pressure  $p_A$  and  $p_E$  are given as inputs to the model.

#### 2.2 Friction modeling

An important part of the model from Eq. (2) is the modeling of the friction term. The friction term characterizes the pressure drop  $\Delta p$  at constant mass flow rate through the tube and is dependent on the Reynolds number Rey according to the Darcy-Weisbach equation /6/ and the relationships given in Fig. 3 (b). It is dependent on the friction factor  $\lambda$ (Rey) which is a common value that has been empirically determined for hydraulic smooth and rough tubes in tables as approximation formulas (see Moody diagram in Fig. 3 (b)). For air flows with a Mach number less than 0.6, these equations are applicable for pneumatic tubes /7/.



Fig 3: Dynamic model discretization schema (a) and friction term modeling with Moody diagram (b).

The important difference to general fluidic transmission line modeling is the explicit consideration of turbulent flow and therefore, nonlinear friction term modeling.

# 3. Model validation for inflation and deflation processes

# 3.1 Experimental set-up and validation procedure

**Fig. 4** presents the test rig built for measuring pressure and mass flow rate at rear ends of a pneumatic tube. The schematic sketch of the test rig shows two measurement procedures whose results are used for a validation of the dynamic model from Eq. (1). In set-up (I) with a known volume at the end of the pneumatic tube, the transient behavior is validated. The tube with diameter D and length L is inflated starting at ambient conditions through the shown directional control valve. Afterwards, the tube is exhausted through the same valve. Pressure  $p_A$ ,  $p_E$  and mass flow rate (or normal volume flow rate)  $m_A$  are measured.



Fig. 4: Test rig for measurements of pneumatic tubes.

The simulation set-up is added by a simple pneumatic volume at the end of the pneumatic tube. The results are discussed in Section 3.2, **Fig. 5**. In set-up (II), pressure and flow rate at either ends of the tube are measured. The set-up is changed by a variable exhaust at the end of the pneumatic tube. The pressures  $p_A$  and  $p_E$  are given as input to the dynamic model (with no volume at the end of the tube) and the two simulated flow rates at rear ends are used for validation purposes. This set-up is used to study the "steady-state behavior" of the tube, respectively the pressure drop over the tube at constant flow rate (here with transient changes between set-points). Both validations are performed for tubes with varying diameters, lengths and small tubes at different supply pressures and with different end volumes. Due to space limitations, the results are not given here.

## 3.2 Results simulation vs. measurements

The dynamic model from Eq. (1) is validated against measurements by using the test rig from Section 3.1. Pressure  $p_A$  is taken as input to the simulation for procedure (I) with additional volume model at end of tube (to simulate pressure  $p_E$ ) and pressure  $p_A$ 

and  $p_E$  for procedure (II) (otherwise a modeling of the control valve with delayed opening behavior has to be performed, but is neglected here).



Fig. 5: Results of validation procedure (I) with D=5.7mm and L=100m.

The parameters that are defined ahead are diameter, length, initial pressure/flow rate and surrounding temperature. The friction factor is calculated by  $f_{nl}$  and a smooth implementation of the formulas acc. to Fig. 3 (b).



Fig. 6: Results of validation procedure (II) with D=5.7mm and L=100m.

The number of discretization elements is fixed to 5. Further simulation runs show that a value up to 10 segments for very long tubes is reasonable; the simulation results change negligible with varying n, parameters such as D and L have more influence on the results. The accordance between the measurements and the simulation is obvious. In Fig. 4, the simulated pressure  $p_{E,s}$  is compared to the measured one  $p_{E,m}$ . The dead

time and delayed pressure rise at the start of the inflation process and the mass flow rate at the beginning of the tube are sufficiently reflected. The error plots show deviations in the transient behavior which are negligible compared to the plots with absolute values. Fig. 5 shows that for small pressure drops (which are reasonable for real applications) up to 0.3 bar, the mass flow rates are reflected perfectly by the model. Hence, the parameter set-up including the friction term modeling is reasonable for long pneumatic tubes.

# 4. Overall system simulation

The choice and dimensioning of pneumatic and electrical drive components is currently done at Festo with the help of the in-company simulation software CACOS<sup>®</sup>. The graphical user interface with selected component libraries for drives, control elements, and accessories is shown in **Fig. 7**.



Fig. 7: User interface of simulation software CACOS® from Festo

Similar to block-orientated simulation programs such as Dymola or SimulationX, the components are connected by ports, each characterized by flow and potential variables (pneumatics: flow rate and pressure, mechanics: velocity and force, electrics: current

and voltage). The connection of elements leads to algebraic equations representing the resulting constraints for flow and potential variables. Each component is described mathematically by dynamic (ODE) or static (algebraic) equations. The states such as position and pressure are calculated by numerical simulation. In compressed air systems, pneumatic tubes are connecting drive components such as cylinders, throttles and valves. Pneumatic tubes have been approximated so far by the mathematical description from Fig. 1. The inner volume of the connecting tube has been added symmetrically to the neighbouring components. The velocity of moving pressure waves has not been of interest so far and has been simply neglected. The reason is that the model errors are negligible for short tubes, huge cylinder rode diameters and/or low cylinder rode velocities. **Fig. 8** shows the discrepancy between the approximation formula from Fig. 1 and the steady-state behavior of the friction term implementation according to Fig. 3(b) for varying pressure drops over a tube.



Fig. 8: Comparison of steady-state pressure drop over pipe

The simulation time in seconds is dependent on the computational capacity of the used computer, the number of implemented ODEs for each tube model, the step size of the DAE solver and the required accuracy of the simulation results. For the dynamical model from Eq. (2), the step size of the ODE solver and the number of ODEs is dependent on the number of spatial discretization segments (Fig. 3). The maximum allowed step size  $\Delta t_{step}$  for the pneumatic (tube) volume V is dependent both on the system dynamics and the number of segments n. It is approximately given by the formula

$$\Delta t_{step} < \frac{1}{5} \frac{V/n}{\sum \dot{Q}}, \quad \sum \dot{Q} = \text{sum of in- and outflowing volume flow.}$$
 (3)

Hence, the step size of the shown dynamic tube model is limited to this step size. Table 1 shows the results of a study on the influence of spatial discretization elements on the model performance, The chosen number of segments results from a reasonable trade-off between number of ODEs, computational speed (related to step size) and model accuracy. The suggestions given are dependent on the length of the tube.

Tube length in [m]	Number of segments	Number of ODEs
< 2m	0, refer to Fig. (1)	0
< 10m	3	6
≤ 100m	5	10
> 100m	10	20

 Table 1: Suggestion for number of spatial discretization segments for tube simulation

Tubes less than 2m are modelled by Fig. (1). **Fig. 9** shows exemplary the comparison of measurement and simulation results under varying spatial discretization elements of the simulation model.



Fig. 9: Simulative evaluation of varying discretization segments n

## 5. Conclusion

Pneumatic tubes are relevant components of pneumatic drive systems with influence on the overall dynamic of the system. The influence of pneumatic tubes is shown and four main dynamic characteristics are identified. The one-dimensional average friction model is chosen for numerical implementation by a vertical line method combined with the control-volume based approach. The friction term is modelled particularly for compressible flow characteristics in tubes. The model is validated with differing tubes and shows good correspondence with the measurements, both for transient and steady-state behavior. The model is shown to be adaptable for different boundary conditions, necessary especially in signal-flow oriented simulation programs. The model works with industry standard parameters, and is able to accurately replicate dynamic and steady-state behavior. Current research is working on the integration of temperature processes in the model equations as high dynamic flows within the tube can lead to temperature changes that cannot be neglected.

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# Nomenclature

<i>C, b</i>	Sonic conductance, critical pressure ratio	m3/(s*Pa), dimensionless
D,L,A	Diameter, length and area of tube	m, m, m2
R, T	Air gas constant, ambient temperature	J/(kg*K), K
p	absolute pressure (A,E: rear ends of tube, H/L: high and low pressure)	Pascal (Pa)
<i>ṁ</i> , Q	Mass flow rate, normal volume flow rate	kg/s, m3/min (Nm3/min)
<i>q, Δр</i>	Pressure ratio, pressure drop	dimensionless, Pa
λ(Rey)	Friction factor, Reynolds number	dimensionless
ho, $ar ho$	density of air, average density of air	kg/m3, kg/m3
<i>v(T)</i>	Kinematic viscosity of air	m2/s