# A Fluid – Structure Interaction model to analyze Axial Balance in External Gear Machines

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# Abstract

This paper presents a novel approach for studying the lubricating gap between lateral bushes and spur gears in external gear machines. Pressure compensated lateral bushes are important elements for efficient operation of an external gear pump or motor, being responsible for functions such as sealing the displacement chambers, and limit the local pressure peaks and cavitation associated with the teeth meshing process. Due to the complexity of creating a dynamic model of fluid film lubrication for this kind of machine, efforts thus far have stopped short of analysing the axial balance of the lateral plates and the hydrodynamic squeeze effect of fluid film lubrication has not been considered. The current study describes an original method of modelling the axial balance of the lateral bushes considering full hydrodynamic and elastohydrodynamic effects coupled to the motion equation of the bushes. The pressure field in the gap is solved using a finite volume solver of the Reynolds Equation. The fluid flow in the lateral gap is fully coupled with the deformation caused in the lateral bushing, which is solved using a finite volume stress solver. Details of the developed solution for solving this complex Fluid – Structure Interaction problem are reported in the paper, and results are presented for a representative gear machine design.

KEYWORDS: external gear machine, elastohydrodynamic lubrication, finite volume method, fluid structure interaction

# 1. Introduction

External gear units (GUs) continue to be widely used in fluid power applications as both hydraulic pumps and motors. While GUs in practical applications have the limitation of being fixed displacement machines, their advantages of being easy to manufacture, low comparative costs and good efficiencies mean that they continue to play a primary

role in many fixed and mobile fluid power applications. Research based on improving the performance (in particular efficiency and reliability) of current GUs thus continues to be of importance. The principle of operation of GUs is simple and easy to understand-(**Fig. 1A**): the fluid is delivered from the inlet port to the outlet port by the rotation of the gears and displaced by the meshing process between the gears (**Fig. 1B**).

Despite the simple principle of operation, the complexity in studying GUs arises from the multiplicity of functions accomplished by each component. In particular, gears are responsible of the displacing action and of the mechanical/fluid energy conversions, while lateral bushes have the primary function of sealing the tooth space volumes (TSVs), but they are also use to establish a proper timing of the connections of the volumes in the meshing zone. In fact, with proper relief grooves (**Fig. 2B**) TSVs are never isolated from the inlet or the outlet port, thus avoiding pressure peaks or cavitation. Through appropriate grooves machined in the outer region, opposite to the meshing zone (Fig. 2B), the lateral bushes permit also to define the location for the transition from low pressure and high pressure for each TSV with gear rotation, thus characterizing the radial pressure distribution acting on each gear (Fig. 1B). Therefore, lateral bushes are a very important component for efficient performance /1-4/.



**Figure 1:** A) GU – exploded view; B) Cross section of a GU, with a qualitative representation of the transition from low pressure to high pressure of each displacement chamber (tooth space volume)

The focus of the present study is the lubricating gap between the gears and the lateral bushes represented in **Fig. 2A**. The design of lateral bushes with proper axial balance is a very important and delicate problem in a GU since it must achieve the goal of sealing the gap, while avoiding excessive shear stresses due to boundary lubrication and wear. On the side of the lateral bushes facing away from the gears (from here on referred to as the *balance side*), there are three regions that are clearly shown in **Fig. 2C**. The pressure on the regions of the balance side of the lateral bushes are axially balanced and have comparable sealing performances over a large range of operating conditions of the GU. Lubricating

gaps in GUs have been studied previously using both standard CFD /5/ applications as well as using applications from lubrication theory /6-8/. The effect of parameters such as operating speed, pressure and angle of tilt of the lateral bushing has also been studied /6-8/. Nevertheless, such studies have always involved an assumption of certain gap height (and/or tilt). Due to the complexity of creating a dynamic model of fluid film lubrication, a prediction of the axial balance characteristics of the lateral bushes and of the hydrodynamic squeeze effect of fluid film lubrication has not been considered.



Figure 2: A) Lubricating gap between lateral bushings and gears; B) lateral bushes considered in this work: side facing the gears (gap side) C) Side facing away from the gears (balance side)

The present authors have been involved in the research challenge of having an omnicomprehensive approach of modelling a GU that is capable of considering all the main phenomena characterizing its operation. The simulation tool named HYGESim (HYdraulic GEar machines Simulator) is the product of this effort. The tool is described in /1,9,10/, and the Fluid Structure Interaction Model (FSI) model presented in this paper is integrated within HYGESim as shown in **Fig. 3**. Preliminary modelling of the lubricating gap with HYGESim was presented in /9-11/. These works did not include FSI modelling of the gap, although micron-level of gap heights were found as realistic for a GU. High pressures in the lubricating gap, along with micron - level gap heights are expected to cause significant solid deformation. In particular, this regime is considered as Elastohydrodynamic (EHD) lubrication. Fully coupled EHD models have been used to study lubricating interfaces in automotive piston cylinder, connecting rod bearing, journal bearings and other interfaces /12, 13/. Numerical improvement of EHD models have also been studied /14/. In axial piston positive displacement machines also, EHD has been found to play a significant role /15/. It is, therefore, anticipated that the pressures generated via the mechanism of deformation can affect the axial balance of the GU lateral bushings. The investigation of this aspect is the main purpose of this work that for the first time includes the axial balance modelling of the lateral bushing in GUs considering full EHD effects coupled to a motion equation of the bushing.



Figure 3 A) Structure of the HYGESim simulation tool B) Detail of the fully coupled FSI model with fluid and structural finite volume submodels



**Figure 4:** A) Showing the meshes for  $\theta = 0^{\circ}$  and  $\theta = 15^{\circ}$  (convention for  $\theta$  indicated in figure with respect to reference TSV in red). Average size of the fluid mesh is ~44500 elements for the case considered C) Boundary conditions is set on the interfaces shown from the HYGESim Fluid Dynamic Model

# 2. FSI – Axial Balance model details

The FSI – Axial Balance models have several submodels that account for the complicated geometry and the interactions of the lubricating gap in GUs.

## 2.1. Automatic dynamic fluid mesh generation and boundary conditions

A fully automatic application was developed in C++ to enable creation of the finite volume (FV) mesh of the fluid domain starting from the CAD drawings of a GU. Firstly, as a pre-processing step, a FV mesh consisting of 6 node prism elements of the gears are created. From this initial FV domain the areas of the grooves are then removed to obtain the lubricating gap domain. **Figure 4** shows two representative meshes for different angles of rotation. The boundary conditions for the lubricating gap domain are set using the results from the fluid dynamic model of HYGESim. The main interfaces on which the interaction takes place are also shown in Fig. 4.

## 2.2. Definition of the lubricating lap geometry and Reynolds equation

The notation used in the model for the lubricating gap of Fig. 2 is shown in **Fig. 5**. Using the gap height information stored at the points T1, T2 and T3 the gap height for an arbitrary point in the domain can be calculated using the following,





The governing equation used to model fluid flows in lubricating gaps is well-known - Reynolds equation /16/. The most general form of the equation is:

$$\nabla \cdot \left(\frac{-\rho h^3}{12\mu} \nabla p\right) + \frac{\rho \nabla h}{2} (\mathbf{v}_t + \mathbf{v}_b) - \rho \mathbf{v}_t \cdot \nabla h + \rho \frac{\partial h}{\partial t} = 0$$
<sup>(2)</sup>

In the geometry being used in the current case (shown in Fig. 5) the velocity of the upper surface,  $\mathbf{v}_t$ , is null since the lateral bushes are stationary. The lower surface corresponds to the lateral surface of the gears and rotates with a velocity  $\mathbf{v}_b = \mathbf{v}_g$ . So the governing equation for the fluid flow in this particular problem is:

$$\nabla \cdot \left(\frac{-\rho h^3}{12\mu} \nabla p\right) + \frac{\rho \nabla h}{2} \left(\mathbf{v}_g\right) + \rho \frac{\partial h}{\partial t} = 0$$
(3)

A finite volume method (FVM) solver for the above equation in the domain of Fig. 5 has been implemented in an application developed in C++ using the open source OpenFOAM /17/ libraries. The pressure is solved for using a very fast implementation of the Preconditioned Conjugate Gradient algorithm, with a Diagonalized Incomplete Cholesky preconditioner. In addition, information about the squeeze velocities (final term in Eq. 3) are also stored at the three points shown in the Fig. 5 and the resultant squeeze velocity at an arbitrary point is given by Eq. 4.

$$\frac{\partial h(x,y)}{\partial t} = x \frac{2\frac{\partial h_{T2}}{\partial t} - \frac{\partial h_{T1}}{\partial t} - \frac{\partial h_{T0}}{\partial t}}{2(d+R)} + y \frac{\frac{\partial h_{T1}}{\partial t} - \frac{\partial h_{T0}}{\partial t}}{2R} + \frac{\frac{\partial h_{T0}}{\partial t} - \frac{\partial h_{T1}}{\partial t}}{2}$$
(4)

#### 2.3. Pressures and corresponding forces on the lateral bushing

The axial balance and structural deformation of the lateral bushes in GUs can be evaluated only through an accurate modeling of the pressures and of the corresponding forces acting on it.

*Pressures & forces acting on the balancing areas:* The balancing areas of the lateral bushes have a high pressure and a low pressure area separated by a seal (which is assumed to be at high pressure). These are highlighted in Fig. 2C. The individual forces and the resultant is calculated accordingly

$$F_{Bal} = F_{HP,Bal} + F_{LP,Bal} = p_{HP}A_{HP,Bal} + p_{LP}A_{LP,Bal}$$
(5)

*Pressures & forces acting on the relief grooves:* The pressures acting on the relief grooves can be assumed to be uniform (only variable with time) at the high pressure or the low pressure. The pressures on the relief grooves are shown in Fig. 2B. The resultant force acting on the lateral bushes due to the relief grooves is given by

$$F_{Rel} = F_{HP} + F_{LP} = p_{HP} A_{HP,Rel} + p_{LP} A_{LP,Rel}$$
(6)

*Pressures & forces acting on the areas corresponding to the TSVs:* The pressures in the TSVs of the two gears are calculated using the fluid dynamic model of HYGESim /1/. A typical distribution of the pressures from the TSV acting on the lateral bushing is shown in **Fig. 6A**. The resultant force is

$$F_{TSV} = \sum_{i}^{z} p_{i} A_{i} \tag{7}$$

*Pressures & forces from the lubricating gap:* The pressures in the gap are calculated by the solution of the Reynolds equation. Since the gap domain is discretized using an FV mesh the resultant force must be calculated using the pressures in the individual cells and also the areas of the individual cells

$$F_{Gap} = \sum_{i}^{nFaces} p_i A_i \tag{8}$$

A typical distribution of pressure in the gap is shown in Fig. 6B.



**Figure 6:** The pressures acting on the lateral bushing from the TSV and Reliefs (A) and the lubricating gap (B, for a constant gap height)

# 2.4. Structural model for the lateral bushing

Elastic deformation of the lateral bushing was modeled using a 3D FVM solver that was developed in a C++ application using OpenFOAM libraries. A 3D FV mesh of the lateral bushing was created using tetrahedral elements (**Fig. 7A**). Several FVM structural solvers have been developed and validated /18, 19/ and in the present model the form discussed in /18/ is used (shown in Eq. 9).

$$\frac{\partial^{2}(\rho \mathbf{u})}{\partial t^{2}} - \underbrace{\nabla \cdot \left[(2\vartheta + \lambda)\nabla \mathbf{u}\right]}_{implicit} - \underbrace{\nabla \cdot \left[\vartheta(\nabla \mathbf{u})^{\mathrm{T}} + \lambda \mathrm{Itr}(\nabla \mathbf{u}) - \left[(\vartheta + \lambda)\nabla \mathbf{u}\right]\right]}_{explicit} = \rho \mathbf{f}$$
(9)

The above equation was solved using an implementation of the Generalized Geometric – Algebraic Multi-Grid (GAMG) algorithm. The implicit and explicit grouping of the spatial derivative terms as shown in Eq. 5 promotes the stability of the algorithm /18/, which was convergent with speeds comparable to commercial FEM software.

# 2.4.1. Deformation calculation using the Influence Operator

The deformation of the lateral bushing is calculated offline, using the influence operator, **E**. Since the solution to the FSI problem requires several evaluations of both fluid pressure and solid deformation this technique leads to an efficient algorithm and has been used previously for EHD models /12-15/. **E** is a matrix that stores pre-calculated deformation of the lateral bushes towards the gap for reference pressure

loads that are applied. In order to generate **E** for the lateral bushing, firstly reference pressure loads are applied on all the constant pressure regions. However, for the face of the bushing that interfaces with the lubricating gap (the gap interface), a complex pressure field is expected (as shown in Fig. 6 A&B), and therefore several components of **E** correspond to the reference pressure loads on each cell face. Individual components of **E** are shown in **Fig. 7** B&C. Having calculated the influence operator for the lateral bushing the deformation for any pressure loads (both on the balance and gap side as discussed in 2.3) can be calculated using,

$$\delta h_i = \sum_j E_{ji} \frac{p_j}{p_{Ref}} \tag{10}$$

Where i, j =  $1,2...,N_p$  are the number of boundary faces on the gap interface of the lateral bushing, and  $p_j$  is the pressure loading, either on the constant pressure regions or from the solution of the Reynolds equation in the lubricating gap.



Figure 7: A) 3D Finite Volume mesh for the lateral bushing B) Component of E for a reference pressure on the HP relief groove and C) reference pressure on a boundary face of the gap interface



Figure 8: Numerical algorithm for the FSI model

# 2.5. Numerical solution algorithm

The solution algorithm proceeds by considering that the lateral bushing achieves static equilibrium at every instantaneous angular position of the gears.

This means:

- The resultant force *F<sub>Res</sub>* acting on the lateral bushing is 0
- The resultant moments on the axes orthogonal to the forces (*M<sub>x</sub>*, *M<sub>y</sub>* for forces aligned along the Z axis) must be 0

Pressure fields satisfying the above conditions are calculated by the Powell's multidimensional root – finding algorithm to set appropriate squeeze velocities,  $\frac{\partial h}{\partial t}$ , at the points T0, T1 and T2. The effect of bushing deformation is also taken into account by performing an FSI loop until both the fluid pressure and solid deformation reach convergence. The squeeze velocities are then integrated (Eq. 12) to find the instantaneous gap heights at the 3 points using a constant time step ( $\delta t$ ). The numerical algorithm is shown in **Fig. 8**.

$$h_n^{\ 0} = h_n^{\ 1} + \frac{1}{2} \left( \frac{\partial h_n^{\ 0}}{\partial t} + \frac{\partial h_n^{\ 1}}{\partial t} \right) \delta t; \ n = 0, 1, 2$$
(12)

# 3. Results

The FSI – Axial Balance model described was used to predict the lubricating gap heights for a 11.2 cm<sup>3</sup> Casappa prototype 12 – tooth GU. The results from a rigid model of the lateral bushing will compared to those given by the FSI model (that considers EHD effects) for the operating condition of: pump speed ( $\omega$ ) = 2000 RPM, pressure difference ( $\Delta p$ ) = 150 Bar and temperature (*T*) = 50 °C.



Figure 9: A) Convergence of the rigid model and B) Lubricating gap heights

**Figure 9** shows the predicted gap height at convergence without considering the deformation of the lateral bushing. The tilt of the bushing (quantified using Eq. 13) is t = 0.31, a positive value signifying that the lowest clearance is adjacent to the HP port.

$$t = \frac{h_{max} - h_{min}}{2h_{avg}} \tag{13}$$

Results from the novel FSI model are shown in **Fig. 10**. It can be noted that the predictions differ both in magnitude and in direction of the tilt which in this case is t = -0.63; the lowest clearance adjacent to the LP port. Thus, it is apparent that the deformation has a significant effect on the axial balance model predictions.



Figure 10: A) Convergence of the FSI model and B) Lubricating gap heights

The gap height magnitudes predicted by the FSI model adjacent to the LP port are very close to the manufacturing tolerance of the parts. Consequently, it would be expected that this part of the bushing can face wear during the operation in the above mentioned conditions. This was in fact observed in experimental tests, and the view of the bushing after the operation of the pump is shown in **Fig 11A**. The large difference in the predictions from the rigid *v*/s the FSI model can be explained by the large deformations that occur due to the high pressures present in the lubricating gap, which in turn cause hydrodynamic pressure generation. The pressure field and the corresponding deformation in the gap are shown in **Fig. 11 B and C**. The high pressure zones corresponds to regions of high deformation that is of the order of the gap heights predicted thereby justifying the use of a model capable of considering EHD effects.



**Figure 11:** A) Wear observed during the operation of prototype B) Pressure in the gap C) Deformation of the bushing in the lubricating gap

# 4. Conclusion

In this study a novel FSI model was presented that has the ability to predict the lubricating gap heights in a GU. In particular, the model is shown evaluate accurately the axial balance characteristics of a GU. As an integrated submodel of HYGESim the FSI model developed was provided very accurate pressure boundary conditions from TSVs, and also an accurate geometrical approach. All the important pressure loads acting on the lateral bushing were evaluated and a novel algorithm developed to account for the EHD effect that arises out of the deformation of the boundary solids in lubricating interfaces. The design of well-balanced GUs continues to be a very delicate design issue, with few design tools currently available to assist in the process and thus far largely dependent on expertise and experience. The present model developed has potentials to be used as a tool to drive designs of lateral bushings, that result in GUs that are well balanced, with low losses as well as low chances of wear.

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# 6. Nomenclature

$A = area, m^2$	F = force, N
f = body force vector, N	I = unit tensor
M = Moment, N-m	R = outer radius of gears, m
d = wheelbase of the pump, m	h = lubricating gap height/film thickness, m
nFaces = number of FV boundary faces	p = pressure, Pa
<b>u</b> = deformation vector, m	<b>v</b> = cartesian velocity vector, m/s
z = number of gear teeth	$\rho$ = density, Kg/m <sup>3</sup>
$\mu$ = kinematic viscosity of working fluid, Pa-s	$\omega$ = angular velocity, rad/s
$\delta$ = boundary mesh resolution, m	$\delta h$ = deformation of the bushing, m
$\vartheta, \lambda$ = Lame's coefficients	
Subscripts:	
Bal = balance	Gap = lubricating gap
HP = high pressure	Rel = relief groove
Ref = reference	TSV = tooth space
Res = resultant	g = gears
b = bottom surface	t = top surface
Acronyms	
CAD = Computer Aided Design	CFD = Computational Fluid Dynamic
FEM = Finite Element Method	FSI = Fluid Structure Interaction
FVM = Finite Volume Method	GU = External Gear Unit
HP = High Pressure	LP = Low Pressure
TSV = Tooth Space Volume	