Unsteady Flow through a Valve Plate Restrictor in a Hydraulic Pump/Motor Unit

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Abstract

Noise is a well known challenge in hydraulic systems. Hydrostatic machines are among the largest noise contributors in a hydraulic system. The noise from the machine originates from flow pulsations at the discharge and suction ports, as well as pulsations in piston forces and bending moments. This article investigates the dynamic behaviour of unsteady flow through a valve plate in an axial piston pump. The proposed extension of the steady state restrictor equation includes a dynamic internal mass term and a resistance. The results from 1D model are validated with a 3D CFD model. Different valve plates' configurations and pump sizes are easily simulated with the two simulation models. The simulation results show very good comparison with experimental tests. The proposed method is verified with a hydraulic pump application but it can probably also apply for original restrictors too.

KEYWORDS: Flow pulsations, fluid power pump, fluid borne noise, non-linear flow

1 Introduction

Pumps and motors are two of the main noise contributors in a hydraulic system. The pump delivers a mean flow with superimposed ripple containing a kinematic and a compressible part depending on the limited number of pumping elements and the finite stiffness of the oil. The flow ripple interacts with the pipe walls and creates a pressure wave which propagates through the connected pipeline system and if the cross-section of the pipes changes, through a valve or other intermittencies which interrupt the pressure wave, the wave will be partially reflected back in the pipeline and a standing wave is created. In addition to the fluid-borne noise, pulsations in piston forces and bending moments create noise as well as cavitation. Cavitation occurs when the pressure falls below a certain saturation level and air is released from the oil. When the pressure subsequently rises,

the air bubbles implode and major fatigue and noise are created. This can occur in very small areas inside the pump and is rather difficult to predict.

In the design phase, it is important to be able to predict the dynamic behaviour of the pump. Development of a new product is faster with a good simulation model, which reduces development costs and shortens time to market. In the early design phase, the simulation is preferably fast and may be connected to optimisation algorithms while in later stages of the design phase a more advanced model is suitable for more exhaustive investigations of phenomena, e.g. cavitation. Regardless of the simulation method the models have to be verified by measurements.

Designing high performance hydraulic machines requires a good, accurate dynamic simulation model. The researchers have investigated the dynamic behaviour in hydraulic valves and discovered that the pressure drop at unsteady flow can not be described by their steady state equation, /1; 2; 3/. In addition, many researchers have found that the inertance on the passageways has an important impact of the dynamic behaviour of components. The dynamic behaviour of a fluid power component is strongly dependent on the hydraulic inductance and hence the inertia of the fluid in the passageway /4/. In /5/ the inertance at the valve plate in an axial piston pump is considered. The authors found that the inertance, particularly in pressure relief grooves, strongly influences the flow pulsation. /6/ shows that inertance affects the flow ripple in vane pumps. This paper takes both the inertance and the unsteady flow into consideration when modelling the flow pulsation created at the valve plate in an axial piston pump.

The motivation of this article is to investigate the reason for some particular oscillations which appear in some pump designs, mainly when the pump creates heavy flow pulsations. This was earlier examined in /5/. The oscillations discussed in this article appear even if the pressure relief grooves is not apparent, which is not clearly verified in /5/. In /7/ the oscillations were explained by the flow spectrum being truncated, i.e. 3 kHz which was used was not sufficient to describe the complete source flow.

The one-dimensional simulation model is made in the free simulation package HOPSAN /8/. The program is built on transmission lines which makes the simulation fast and suitable in the early design phase. The results in section 3 compare the 1D simulation model to a high fidelity three dimensional (3D) computational fluid dynamics simulation model (CFD), /9/. The flow simulations are validated by the anechoic termination method, /10/. The method is based on there being no reflective waves in the system. This means that pressure ripple is the same in the whole system if viscous friction in the pipe can be neglected. The method is simple but does not give both the source flow and source impedance from the pump. The method is suitable for pumps with high impedance outlet channels, i.e. small volume and leakage.

The article uses an axial piston pump, both bent axis and swash plate types, as the object. However, the modelling approach can be applied in other pump models too. The used object has a clearly unsteady pulsating flow though a variable restrictor, which makes it an obvious component to apply the technique to. The approach is probably also suitable for ordinary restrictors.

Section 2 begins by explaining how the 1D simulation is built for pump flow pulsations and continues with the extension of the steady state orifice equation. The 3D model is then introduced. The results from both simulations and measurements are presented in section 3. The article ends with a discussion section and some final remarks.

2 Simulation methods

Simulation can be performed with different levels of complexity. The user's purpose with the model varies: Study parameter variation in new or improved components; investigate behaviours of a component or system; study failures and problems in already existing features. In the pump case, the level of complexity depends on the purpose of the simulations. In a system level investigation, a typical pump flow ripple is more important than how the flow ripple is created. In /11/ a typical pump flow ripple is simulated with just two parameters. However, when the pump itself is the feature of interest, a more complex model is needed. In section 2.1, the 1D simulation model is explained, including the proposed method to model the unsteady flow through a restrictor. Section 2.2 contains a short description of the 3D CFD model.

2.1 1D simulation model

A one-dimensional model is made in the simulation program HOPSAN, /8/, which is mainly used for hydraulic simulations. The components are implemented with transmission line theory, TLM, e.g. /12/. The pump model used in this paper originates from /13/ and further developed in /14/. HOPSAN uses distributed model structure which makes the calculations very effective due to allowableness of distributed solvers and numerically stiff due to the finite signal propagation speed. The pump model is distributed to individual components, i.e. each cylinder in the pump, two orifices per cylinder representing opening to high and low pressure port respectively, etc. In this way the system is kept close to reality. This is illustrated in figure 1(a).

Figure 1(b) illustrates how the flow is modelled in Hopsan. Each cylinder's capacity is modelled with a single volume. The size of the volume and the bulk modulus changes during simulation according to the piston motion and the pressurisation. The volume size and bulk modulus represent the compressible flow ripple and are modelled as equation (1) while the piston motion represents the kinematic flow ripple.

$$q_{comp} = \frac{V}{\beta_e} \frac{dp}{dt} \tag{1}$$



(a) The distributed pump model structure where each component in the pump have the own model component with the required equations.



(b) The flow modelling in Hopsan. Kinematic flow modelled by piston motion and compressible flow modelled with a capacitance.

Figure 1: Illustration of the modelling technique for the 1D model in HOPSAN.

V is the cylinder volume, β_e is the effective bulk modulus of the oil and $\frac{dp}{dt}$ is the pressure change inside the cylinder.

In previous measurements of flow, e.g. /7/, it was found out that the already developed axial piston pump model in Hopsan does not fully describe two-microphone method /15/ was used for these measurements. In some measurements of a particular feature and operation conditions there is an oscillation of the flow which is not earlier clarified in the under certain simulation model. The measurements were made up to 3 kHz and the additional frequency is above that frequency. The explanation for the poor results for in particular zero-lapped valve plate was that the flow spectrum was truncated, i.e. 3 kHz was not sufficient to describe the complete source flow.

The variations in the bulk modulus due to pressure changes from inlet to outlet and also the air content are modelled according to the tangent value, equation (2).

$$\beta_e = \frac{\beta_{oil}}{1 + \frac{x_0}{\kappa_p} \frac{\beta_{oil}}{1 - x_0} \left(\frac{p_0}{p}\right)^{\frac{1}{\kappa}}}$$
(2)

The pressurisation and depressurisation in the cylinders occurs very fast and can therefore be assumed to be adiabatic. The cavitation is protected by limiting the lowest pressure in each cylinder. The method is assumed to be satisfactory if cavitation is suppressed.

The valve plate opening and closing to the ports are modelled with the steady state equation for a turbulent restrictor according to equation (3).

$$q = C_q A \sqrt{\frac{2}{\rho} \Delta p_{valve}}$$
(3)

 Δp_{valve} is the pressure drop over the valve. C_q is the flow coefficient and is known to be a function of the Reynolds number and the area difference between the orifice and the

pipe. The value is very difficult to estimate in the pump environment and the standard value for turbulent hole orifice of 0.60, /16/, is therefore chosen. The equation is not completely correct at unsteady flow through the orifice but the approximation is justified in most cases. The equation is not quit valid for non-linear dynamic flow. The dynamic flow through an orifice has been studied earlier, e.g. /1; 3/; however, no clear modelling technique has yet been explained.

The proposed extension of the steady state restrictor equation includes a dynamic internal mass term which is illustrated in figure 2(a). The pressure indexes are $p_1 - p_2 = \Delta p_{orifice}$ and $p_2 - p_3 = \Delta p_{mass}$. figure 2(b) and 2(c) shows the jet flow which creates the internal mass at the valve plate opening.



(a) Illustration of the steady state orifice and the internal mass model.

(b) Valve plate opening restrictor area. (c) Internal mass of the jet flow from the valve plate opening.

Figure 2: Modelling of the internal mass dynamic behaviour.

The pressure drop due to internal mass can be calculated as equation (4). This equation neglects the oil's compressibility. The pressure drop includes a resistive and an inertial term /17/.

$$\Delta p_{mass} = L \frac{dq}{dt} + Rq \tag{4}$$

By using equation (3) and (4), the total pressure drop over the valve $p_{tot} = p_3 - p_1$ is calculated as:

$$\Delta p_{tot} = L \frac{dq}{dt} + Rq + \frac{q^2 \rho}{2C_q^2 A^2}$$
(5)

The inertial term is represented by the inductance L which is commonly calculated as equation 6.

$$L = \frac{m}{A^2} = \frac{l_e \rho}{A} \tag{6}$$

where *m* is the internal mass created by the volume of the jet flow in the orifice at the valve opening, figure 2(c). For circular restrictors, the term l_e represents the effective length of the restrictor and is commonly expressed as.

$$l_e = l_{orifice} + cd_{orifice} \tag{7}$$

 l_e is not easy to determine. In /18/, the circular restrictor should be lengthen by $\frac{\pi}{4} < c < \frac{8}{3\pi}$ due to jet stream effects. These values are for a single expansion or contraction while in the valve plate restrictor the end correction is for a double-ended restrictor and hence twice as big. The value varies due to the length of the restrictor and the value is developed from an infinite pipe wall diameter. In /19/ the effective length is numerically determined for zero mean flow in three different orifices; circular orifice, spool valve orifice, and poppet valve. The inductance is independent of the mean flow and only the geometric of the orifice and the fluid density. This was proved in /20/ with CFD simulations while in /19/ the author is not entirely convinced. The values can not be applied directly to the restrictor in the valve plate, hence the opening is not a circular orifice, the dynamic flow is not considered, and also the none-zero mean flow may have effects on the inductance. The effective length is calculated according to equation (8) with $l_{orifice} = 0$ and $c = \gamma$.

$$l_e = \gamma d_h$$
 where $d_h = \frac{4A}{\Theta}$ (8)

 d_h is the hydraulic diameter of the restrictor. According to calculations made in /19/, the factor γ should decrease by the level of contraction. However, due to the complexity of deciding the effective length, γ assumes a constant value.

R is the resistance and is obtained from derivation of the whole system, i.e. equation (5). The value of *R* become a function of *A* and *m* according to equation (9).

$$R = \frac{\dot{m} - \frac{m\dot{A}}{A}}{A^2} \tag{9}$$

In /21/, a more general derivation of a dynamic restrictor is shown including resistance.

In /3/, measurements of unsteady flow through a sharp-edge orifice are performed. The paper considers the question of whether the model's steady state characteristics such as equation (3) can be used to describe a dynamic flow through the orifice. The unsteady characteristic of non-linear pressure drop in a restrictor varies with a delay when the flow rate is changed. It is assumed to take some time for the turbulent jet flow to change structure after a flow change. The pressure drop can be explained mathematically by the three different kinds of pressure drops. The parameter is estimated from measurement data. In this article, the measurements presented in /3/ are used to verify the proposed method and there is very good accuracy between the theory with internal mass and the parameter value in pump simulations at infinite pipe diameter d_p . γ is dependent on the pipe diameter and hence the value is difficult to compare directly. Also, the damping factor can not be found with the information in the article.

2.2 3D CFD model

The 1D Hopsan simulation model is compared with a CFD model which is created in the commercial program Pumplinx from Simerics, Inc /9/. The code builds on Navier Stokes

formulation of a controlled volume. The simulation program is mainly built for positive displacement pumps and motors. The program has been tested for different types of pumps/motors, mainly from a cavitation point of view, e.g. /22; 23; 24/. A cavitation model from /25/ is implemented; however, the inlet pressure when comparing the 1D model and the 3D model is chosen high to avoid the mismatch as much as possible for the inlet flow results. The comparison for the outlet flow pulsations is not affected as long as the inlet pressure is slightly boosted, 2-3 bar.

With the 3D model, the pump can be easily reshaped and tested in a simulation environment, which is similar to real measurements. However, simulations are always simulations and the results should be treated thereafter, i.e. with caution.

3 Results

To verify and validate the 1D model with the internal mass included, several different measurements and simulations are performed with various operation conditions and pump features. Three different valve plates, zero-lapped, pressure relief groove and precompression angle, are used to illustrate the models' different behaviour. *Zero-lapped* valve plate implies that right at the bottom dead centre, the cylinder closes to the low pressure kidney and opens up to the high pressure kidney, i.e. no pre-compression of the fluid before entering the high pressure kidney. The *pressure relief groove* valve plate has a groove at the entrance to the high pressure kidney and is designed to create small flow pulsation at the operation conditions used in this article. The *pre-compression* valve plate has a rather large pre-compression angle, which implies a big pressure overshoot. All the results show one period, i.e. $2\pi/z$, where z = 7 in the simulated example.

The results from simulating the source flow with and without the internal mass are shown in figure 3 when zero-lapped valve plate is used. The major difference between simulations with and without internal mass is the pulsating flow ripple when the zero-lapped valve plate is used. The increased oscillation can be seen in the frequency domain by increased amplitude at about 4 kHz. The momentum of the reverse flow into the cylinder is high and gives a high excitation of the oscillations.



Figure 3: The 1D model when the cylinder capacity is simulated with a volume and no internal mass (dotted line and bars) and with internal mass (solid line and dots).

Figure 4 shows typical results for pump simulations including internal mass compared

with the CFD model. The valve plate is of zero-lapped design.



Figure 4: Internal mass simulation with 1D model compared to the 3D CFD model, dotted lines and bars and solid lines and dots respectively. The rotational speed is 1000 rpm and outlet pressure 100 bar on the top row while the bottom row shows an operating condition of 2,000 rpm and 200 bar.

The 1D model is also verified with measurements, shown in figure 5. All three valve plates are used for verification. The flow is rather undamped in the measurements for zero-lapped valve compared with simulations. The measurement system was perhaps not completely reflection-free. Some discontinuities such as variations in the pipe cross-section area may exist. The attenuation in simulation is lower than in measurements. The reduced oscillation when pre-compression angle and pressure relief groove are used can be clearly seen. The zero-lapped valve plate gives high oscillations compared to the other valve plates where the pressure build-up is smooth. The pre-compression valve plate builds up too much pressure and when the cylinder connects to the high pressure kidney the excitation energy goes out to the port volume, which damps the oscillations.



Figure 5: Internal mass simulated in Hopsan compared to anechoic termination measurements, dashed line and solid line respectively. The rotational speed is 2,000 rpm and outlet pressure 200 bar.

Different displacements of the pump are simulated with the 1D model and the 3D model. The normalised source flow is shown in figure 6. The oscillation frequency is slightly reduced. However, the two different simulation models have similar behaviour. The damping is in general higher in the 3D CFD model. The oscillation frequency is \approx 2600 Hz for the large pump (200cm³) while the frequency increases to \approx 6500 Hz for 8cm³ at 2000 rpm and outlet pressure at 200 bar.



Figure 6: Normalised source flow for increasing displacement, i.e. left to right 8 cm³/rev, 40 cm³/rev and 200 cm³/rev. Upper row shows the 1D model, the lower row the 3D CFD model. The rotational speed is 2000 rpm and outlet pressure 200 bar. Note, the 40 cm³/rev pump drives with a lower inlet pressure.

4 Discussion

The choice of simulation model is dependent on the purpose of the simulation, except that all the models should be accurate for the investigated phenomena. A general and functionality study of new or improved existing features, especially parametric investigation in combination with an optimisation algorithm, should be fast and simple to be convenient to use. In later stages of the development phase and even for already marketed products, a more exhaustive investigation of the flow dynamics inside the cylinders and pipes may be needed and a heavier calculated simulation model is justified. As computer hardware develops, it is possible to refine the simulation model and more detailed studies are possible. However, the calculation time is still an important restriction when simulation software is chosen. It is of importance to be able to simulate advanced processes like source flow in a simple and fast manner.

The less attenuation that the 1D model shows may be due to the calculation of the resistance. The resistance is calculated as one-dimensional, which may make it smaller than in reality. The 3D CFD model as well as measurements includes additional phenomena which may increase the damping. Flow losses are one phenomenon which is not completely included in the 1D model. There are small differences in the frequency of the oscillation between the measurement and the simulation models. This may be caused by the simplified calculation of the inertance and thus the effective length. The mean flow may also have some impact on the oscillation. In the measurements, the frequency of the oscillation may be reduced due to external system which can not be fully eliminated in the measurements. However, both the divergence in frequency and damping are of less importance and the major behaviour is accurately simulated. The method gives very good accuracy for scaled pumps. The decrease in frequency when the pump displacement increases is accurately simulated. The oscillated frequency for a big pump provides oscillation at a significant frequency with relatively high energy content.

The proposed model with internal mass, calculated from the jet flow from the orifice, improves the accuracy at larger pressure derivatives, e.g. zero-lapped valve plate and similar when the superimposed flow appears. A bigger pump is also simulated, a scaled version of the smaller pump, and the 3D model and 1D model are comparable. No major differences can be seen for a well designed valve plate between simulations with and without internal mass, which implies that previous investigations have been accurate. The internal mass approach is also compared to the measurement results in /3/ with very good accuracy. The models are verified with measurements at reflection free condition. The study shows the importance of adding inductance and resistance when simulating dynamic behaviours.

5 Conclusion

This paper explains the superimposed flow pulsations that occur in some pumps additional to the kinematic and compressible flow pulsations. The proposed method adds a non-linear dynamic term to the steady-state restrictor equation. The 1D model in the simulation program Hopsan is compared to a 3D CFD model of the pump. Also, source flow measurement is used to validate the simulation model. An unsteady flow through a restrictor creates a flow ripple which can be modelled by the inductance and resistance of the jet flow. The damping is satisfied by the derivation of the flow equation. The inductance is dependent on the hydraulic diameter and an end correlation parameter γ , The value is between 1.3 and 1.5 of all tested pump configurations. The proposed approach of adding non-linearity for dynamic flow in the valve plate opening in an axial piston pump can most probably also be used in ordinary restrictors. The example with a pump in this article has an obvious unsteady flow through an orifice.

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6 Symbols

Α	Restrictor area	[m ²]
C_q	Flow pressure coefficient	[-]
С	Length correction factor	[-]
d_h	Hydraulic diameter	[m]
L	Hydraulic inductance	[Ns ² /m ⁵]
l_e	Effective length	[m]
т	Internal mass	[kg]
р	Pressure	[Pa]
q	Flow	[m ³ /s]
R	Resistance	[-]
V	Volume	[m ³]
<i>x</i> ₀	Fraction of free air	[-]
Z.	Number of pistons	[-]
β_e	Effective bulk modulus	[Pa]
β_{oil}	Oil bulk modulus	[Pa]
γ	Length correction factor	[-]
к	Polytropic exponent	[-]
ρ	Density	[kg/m ³]