Experimental and Theoretical Studies of the Displacement and Bending of a Hydrodynamic Supported Idle Spindle of a Three-Spindle Screw Pump

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Abstract

The displacement and elastic bending of a hydrodynamic supported idle screw of a 3spindle screw pump is measured by a set of inductive sensors with a resolution of less than a micro meter. The thus gained experimental results serve to validate a coupled fluid and structure model (FSI) of the pump developed by the authors. The idle screw is modeled as a Bernoulli beam interacting with a hydrodynamic lubrication film. The research task is to predict the operation limit of the screw pump. In fact that research task is nearly reached by our approach.



Figure 1: 3-spindle screw pump, Leistritz Pumpen GmbH

1. Introduction

Screw pumps are kinematic pulsation free positive displacement pumps. The hydrodynamic supported sidewise idle spindles are driven by engagement of the center

drive spindle (**Figure 1**). In unpropitious operation points of low rotating speed, low viscosity and high pressure difference contact between the idle spindle and the casing may occur. It is state of the art to experience this operation limit experimentally. The aim of the present research is to improve the physical understanding of the problem and finally to predict the operation limit on truly physical grounds.

2. Experimental setup

Former experimental studies by the authors leaded to the conclusion that the movement of the idle screw can't be understood as a rigid body movement. Bending aspects have to be taken into account /3/. Therefore a set of four inductive distance sensors was placed along the idle spindle axis. The axial distance of the sensors is equal to the pitch of the idle screw. The sensor head is embedded in a magnetic inert sealing elastomer plug, which is shaped to the casing contour. The sensor and the plug are hold in position by a brass screw (**Figure 2**).



Figure 2: Axial sensor positions

For horizontal and vertical displacement measurement the system is applied in two perpendicular measurement planes. (**Figure 3**).



Figure 3: Coordinate systems, view from suction side

Due to the fact that an absolute changeless calibration of the center of the screw to the center of the housing is impossible, the origin of coordinates is defined by an initial point of operation, which has to be reproduced every measurement series. Experience shows that the measurement results are reproducible.

3. Hydrodynamic mode

The pressure and shear load, p and $\vec{\tau}$, on the idle screw is modeled by 2-dimensional lubrication theory, given by Reynolds equation

$$\nabla \cdot \left(\frac{h^3}{\mu} \nabla p\right) = 6\nabla \cdot \left(h\vec{U}_1 + h\vec{U}_2\right) + 12\frac{\partial h}{\partial t}$$
(1)

and surface tangential stress vector (we call it shear equation)

$$\vec{\tau} = \frac{\mu}{h} \left(\vec{U}_2 - \vec{U}_1 \right) - \frac{h}{2} \nabla p, \tag{2}$$

with gap height *h*, dynamic viscosity μ , spindle wall speed \vec{U}_1 , opposite wall speed \vec{U}_2 , time *t*, and surface normal \vec{n} , leading to the stress vector on the spindle surface

$$\vec{t} = -p\vec{n} + \vec{\tau}.\tag{3}$$

Figure 4 shows the pressure deviation of a standard operation point with a dynamic viscosity μ of 5 mPas, a rotating speed of 3000 rpm and a pressure difference Δp of 39 bar, on the left side on the 2D-projection of the geometry, on the right side on the spindle surface. Leakage between drive and idle spindle is not taken into account,

which is reasonable for having rotor dynamics in focus and not volumetric efficiency of the pump $\frac{1}{2}$.



Figure 4: Pressure distribution at the standard operation point

We take advantage of the linearity of the Reynolds and shear equation (Eqn. (1), (2)) in pressure p, stress vector $\vec{\tau}$ and relative speed. Hence, the influence of the pressure difference, rotating speed, and lateral spindle displacement is treated separately followed by a superposition.

The homogeneous solution of the Reynolds equation (1) is scaled by the pressure difference with the suction pressure p_0 added:

$$p_1 = p_1(\Delta \tilde{p} = 1, \varphi, \xi_1, \xi_2) \,\Delta p + p_0. \tag{4}$$

Unfortunately the pressure distribution is a nonlinear function of the rotational angle φ and the lateral displacements ξ_1 and ξ_2 . This is due to the non-linear influence of the gap height *h*. **Figure 5** shows the distribution of p_1 (Eq. (4)), i.e. the solution of the Reynolds equation (1) for vanishing right hand side.





Figure 5: 1st fundamental solution, influence of pressure difference

The most important inhomogeneous solution of Eq. (1) scales with angular speed Ω and viscosity μ :

$$p_2 = p_2 \left(\tilde{\mu} \widetilde{\Omega} = 1, \varphi, \xi_1, \xi_2 \right) \mu \Omega.$$
(5)

Figure 6 shows the result for an excentric placed idle spindles (the excentricity is in the ξ_1 -direction, see Fig. 3). The pressure distribution typical for hydrodynamic lubrication is predicted (journal bearing effect) by our simulation results.



Figure 6: 2nd fundamental solution, influence of rotating speed

Further fundamental solutions are given by displacement speeds $\dot{\xi}_1$ and $\dot{\xi}_2$ in the lateral directions shown in Fig. 3.

4. Bending line model



Figure 7: Line load diagrams for $\varphi = 0^{\circ}$ and $\varphi = 90^{\circ}$ and simplified for bending line model

From the simulation results line load functions the line load components q_i are obtained with the 1 and 2 direction as shown in Figure 3. The load distribution vs. the spindle length coordinate is shown in Figure 7 for different rotation angles. Although the distribution of the line loads depends on the rotational angle, the resulting lateral force in the ξ_2 -direction is independent of the rotation angle. The lateral force in the ξ_1 direction vanishes (see dark black line in Figure 7). The peak of the line load in the ξ_1 direction at the discharge side (right side of the axis in Figure 7) is due to the design of the sealing system seen in Figures 1, 4, 5. To simulate the elastic deflection of the idle spindle simplified model the load shown in Figure 7 right is used.

With that load distribution the deflection follows from Bernoulli's beam equation in coordinate direction i

$$(EI_i\xi_i^{\prime\prime})^{\prime\prime} = q_i, \tag{6}$$

and its boundary conditions at both spindle ends

$$EI_i\xi_{i\ end}'' = (EI_i\xi_{i\ end}'')' = 0.$$
(7)

The line load is given by the pressure difference line load q_{iP} and rotating speed line load q_{iI} representing the journal bearing effect.

$$q_i = q_{iP} + q_{iJ}(\xi_j, \mu, \Omega). \tag{8}$$

The fluid-structure-problem was solved numerically and iterative, were the structure is described by Eq. (6) and the fluid mechanics by Eq. (1).

5. Results

Due to the fact that an absolute calibration is not possible with the measurement system, the displacement is referred to an operation point of 2 bar, 1500 rpm and an oil temperature of 30°C, resulting in an dynamic viscosity of 5 mPas. For three test series, the spindle elastic displacement is shown in **Figure 8**. 8a shows a series that was done with a constant rotating speed of 1500 rpm and various pressures. In the ξ_{1M} -direction the idle spindle moves towards the casing wall with increasing pressure. In the ξ_{2M} -direction a cardanic displacement in combination with a strong bending can be seen. The simulation result in Figure 8b reproduces the measurement results qualitatively.

Another measurement series was done at 1000 rpm. The observed behavior is nearly identical to the at 1500 rpm. Those measurements were done only up to a pressure difference of 40 bar with regard to operation limits of the pump within this fluidic hydraulic oil. A third measurement series was done at 30 bar and 1500 rpm. By heating the oil the viscosity declined. Here as well the basic behavior could be reproduced by simulations /6/.



6. Summary

For the first time it is seen that the spindles of a 3-spindle screw pump are not rigid but show an elastic bending. By a fluid-structure-approach we could model the rigid and elastic displacement of the idle spindle with a sufficient accuracy to predict the operation limits of a screw pump. Most important is the efficiency of our method, which allows predicting the operation limit already in an early development phase.

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8. References

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9. Nomenclature

p	pressure	Ра
Δp	pressure difference	Ра
p_0	suction pressure	Ра
h	gap height	m
μ	dynamic viscosity	Ра
\vec{U}_1	spindle wall speed	m/s
\vec{U}_2	opposite wall speed	m/s
t	time	S
$ec{ au}$	projection of the stress vector, shear vector	Pa
ť	stress vector	Ра
\vec{n}	spindle surface normal	1

φ	rotation angle	rad
Ω	angular speed	rad/s
ξ	displacement	m
ξ_1	horizontal displacement	m
ξ_2	vertical displacement	m
Ε	elastic modulus	Ра
I _i	geometrical moment of inertia	m ⁴
q_i	line load	N/m
q_1	horizontal line load	N/m
<i>q</i> ₂	vertical line load	N/m
q_{iP}	pressure indicated line load	N/m
q_{iJ}	journal bearing line load	N/m